Indian Statistical Institute, Bangalore
M. Math. Second Year

Second Semester - Simple Groups and Geometry

Mid-Semester Exam   Duration: 3 hours   Date : March 06, 2015

Note: Answer all questions. Your answers should be *complete and precise*. Show your
rough work separately.

Instructor: NSN Sastry   Max Marks: 100

1. (a) Define a t - transitive group. Give an example of a 2 - transitive group which is
   not 3 - transitive. Justify.   [4+6]

   (b) Let X be a transitive G - set. Show that the induced natural action of G on
   $X \times X$ is not transitive, but there is a bijection between the G - orbits in $X \times X$
   and G_x - orbits in X. Here, $x \in X$ and $G_x = \{g \in G : g(x) = x\}$.   [3+12]

   (c) Define the equivalence of two G - sets. Show that the set of all transitive G -
   sets is equivalent to the set of all G - sets of left cosets of subgroups of G. [5+10]

2. (a) Define a projective plane.   [4]

   (b) Show that there is a bijection between the sets of points incident with any two
   lines of a projective plane.   [4]

   (c) i. Find the number of lines of a finite projective plane.
       ii. Show that, if A is a set of points of a projective plane of order n with the
           property that each line meets A in at most 2 points, then $|A| \le n+2$. [4+6]

3. (a) Give an example of a projective plane of order q, q a prime power. Verify.

   (b) Prove that a projective plane of order 3 is unique.   [7+15]

4. Determine   the   conjugacy   classes,   and   the   size   of   each   class,
   of S_6.   [20].

— o o o —